Optimal stock-out risk for a component mounted on several assembly lines in case of emergency supplies

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Abstract: *This article focuses on the calculation of the optimal stock-out risk for a component, which is used by alternative modules mounted on several assembly lines. The studied context is a supply chain dedicated to the mass production of highly diversified products, which is common in the automotive industry. The Material Requirement Planning (MRP) approach is adapted for the monitoring of this chain; however, the distance between the production units leads to mix between production to stock and production to order for the component of interest. To prevent stock-out propagation along the downstream part of the supply chain, use of an emergency supply is triggered prior to its occurrence. The definition of the optimal safety stock and the associated optimal stock-out risk, are based on a mono-period model that considers the cost of a safety stock and the costs incurred by the emergency supply (transportation and production). The analytical solutions that are dependent on these costs are illustrated in this study.*

Keywords: Stock-out risk, Emergency supplies, Safety stock, Supply Chain, Customized mass production.

1. INTRODUCTION

In this article, we focus on the definition of the optimal stock-out risk for an order-up-to-level supply policy. We examine the particular context of the mass production of highly diversified products in which component requirements are supplied for the use of alternative and optional modules on final assembly lines; their overall production is deemed stable and predictable.

Supply Chains (SCs) dedicated to the mass production of highly diversified products are characterized by a certain geographic dispersion of production facilities well known in the automotive industry. In this context, the production is driven by several assembly lines that are geographically remote and whose diversity is mainly ensured by alternative modules (engines, gearboxes, etc.) that are mounted on multiple workstations in a final assembly line. Each workstation is dedicated to a different set of alternative modules, of which one must necessarily be mounted on the finished product that passes through this workstation. An alternative module can be used by many assembly lines and belongs to several alternative sets of modules; each set is specific to an assembly line. Optional modules (sunroof, air conditioning, etc.) are considered as particular alternative modules. Periodic production levels of final assembly lines are stable in the short term or their evolutions, known.

With an established daily production for each line on a horizon of several weeks, the demand of systematically mounted components and of the components they use is certain. In the absence of uncertainty on quality, lead-times and production, the management of this type of flow is beyond the scope of our study.

The production monitoring of alternative modules—and the components they use—is complex. We consider the classic scenario where customer orders to suppliers are delivered simultaneously with similar periodicity. This operation mode is that of the MRP that determines periodically and consistently the production launch of various references of the Bill Of Material (BOM) to ensure compliance with the requirements of the Master Production Schedule (MPS) which derives the production of all productive units of the SC.

Recently, Giard and Sali (2012) and Sali (2012) proposed an adaptation of the MRP approach to control the production of components manufactured in remote units of an upstream SC, which is dedicated to the mass production of highly diversified products. For this type of SC, the requirements of the MPS, for pulling the production of components and alternative modules, are specified at the BOM level corresponding to the alternative modules known as the planning BOM. Over the frozen horizon, these requirements are unknown and can be represented by random variables that are used to determine the safety stocks at different levels of the SC. In this two researches, the accepted stock-out risk is not issued from an economic trade-off and no rule is given to specify its level.

We focus in this study on the economic analysis that should be used to define the optimal stock-out risk when an emergency supply is triggered systematically to prevent the propagation of the stock-out along the downstream part of the SC. In the second section, we describe how to define the problem of emergency supplies. In the third section, we present a model of the problem and the resulting analytical solutions.

2. PROBLEM POSITIONING

In mass production of highly diversified products, the variety of finished products is so great that the MPS has to be defined at the BOM level of alternative modules, which are limited in number.

The requirements of systematically mounted components are known in advance. Thus, these components are beyond the scope of our study as explained previously.

The requirements of alternative modules for periods that are covered by the frozen horizon H_F^l of an assembly line l are known. The frozen horizon delimits what can be produced to order in the upstream SC. The remoteness of the production units in global SCs and the heterogeneity of the frozen horizons associated with the assembly lines lead to an adaption of the MRP approach; this allows mixed make-to-order (MTO) and make-to-stock (MTS) productions. Such adaptation of the MRP is developed by Giard and Sali (2012). We summarize the analytical results presented in their article (§ 2.1). In that study, the order-up-to-level, which is used to address the uncertain part of the demand, is defined using an arbitrarily defined stock-out risk. The determination of the stock-out risk may be an economic trade-off between the cost of emergency supplies and the cost of holding a safety stock. The data used for this arbitrage are detailed (§ 2.2). In section 3, the construction of a general model for decision making is discussed.

2.1. Procurements in a revisited MRP by mixing MTO and MTS

We refer to the results obtained in (Giard and Sali, 2012) and generalized in (Sali, 2012) to consider the potential use of a single component by several alternative modules. The application in cascade of the BOM explosion leads to find a_{ik} units of the component i , which belong to the level n of the BOM, included in one alternative module k belonging to the set $\mathscr{E}_i^l$. $\mathscr{E}_i^l$ is the set of exclusive alternative modules used in the assembly line l that requires the component i .

Moreover, in the MPS, the application in cascade of the lead-time offset mechanism leads to a lag λ_{ik} between the period t of production launch of a reference unit i and the period $t+\lambda_{ik}$ of the requirements of the module k in the MPS. This causes binding of the Gross Requirements (GR_{it}) of a reference i (level n of the BOM) at time t to the requirements $MPS_{kt'}^l$ of the module k (level 1 of the BOM) mounted on the assembly line l at time $t > t'$. This link is different from the classical link that binds the gross requirements of a component i with the planned orders of the references j_i (of level $n-1$ of the BOM) that use that component.

When the demand is certain, the stocks are useless and GR_{it} is equal to the Net Requirements (NR_{it}) and the Planned Order ($PO_{i,t-L_i}$), where L_i is the lead time of the component i . These values are related to the MPS requirements of the final assembly lines by equation (1).

$$PO_{it} = NR_{i,t+L_i} = GR_{i,t+L_i} = \sum_l \sum_{k \in \mathscr{E}_i^l} a_{ik} \times MPS_{k,t+\lambda_{ik}}^l \qquad (1)$$

Beyond the frozen horizon H_F^l of the assembly line *l*, we only know the demand structure recorded in the planning BOMs. In this case, the coefficients of the planning BOMs, which are related to the alternative modules mounted on a workstation of the assembly line, are considered probabilities of use for these modules.

The requirements of the MPS of the assembly line l for the alternative module[1] k in the period $t' > H_F^l$ becomes a random variable $X_{k,t'}^l$. This variable follows a binomial distribution where the number of events corresponds to the number of units of finished products that are assembled on the line l during a review period, and the probability of occurrence of the event is the coefficient c_k^l of the planning BOM associated with the alternative module k mounted on the line l .

$$GR_{i,t+L_i} = \sum_l [\sum_{\mathscr{E}_i^l | \lambda_{ik} < H_F^l} a_{ik} \cdot MPS_{k,t+\lambda_{ik}}^l + \sum_{\mathscr{E}_i^l | \lambda_{ik} \geq H_F^l} a_{ik} \cdot X_{k,t+\lambda_{ik}}^l] \qquad (2)$$

This generalization is essential if one wants to plan the production of remote assembly lines dedicated to the mass production of diversified products with an MRP approach. The Planned Order PO_{it} calculated at the beginning of the period t and delivered at the beginning of the period $t+L_i$ is equal to the certain requirements generated by the part of the MPS covered by the frozen horizon ($\sum_l \sum_{k \in \mathscr{E}_i^l} a_{ik} \times MPS_{k,t+\lambda_{ik}}^l$) plus the difference between the order-up-to level $R_{i,t+L_i}$ and the stock position when making decision. We note OHB_{it} the One-Hand Balance, which is the stock physically held in period t .

[1] If a component i is required by several alternative modules on the workstation with the same coefficient a_{ik} and for the same period, it is necessary to work with a fictitious module k' which regroups that subset of alternative modules. The coefficient of planning BOM for this fictitious module is the sum of the coefficients of modules included in this subset. This allows us to generalize the approach of considering the commonality of components used by several alternative modules in the same assembly line.

$$PO_{it} = \sum_{l} \sum_{k \in \mathscr{E}_i^l} a_{ik} \times MPS_{k,t+\lambda_{ik}}^l + R_{i,t+L_i} - [OHB_{it} + \sum_{h=0}^{h=L_i-1} PO_{i,t-L_i+h} - \sum_{h=0}^{h=L_i-1} \sum_{\mathscr{E}_i^l | \lambda_{ik} < H_G^l + h} a_{ik} \cdot MPS_{k,t+\lambda_{ik}-h}^l] \quad (3)$$

The order-up-to-level $R_{i,t+L_i}$ is the fractile associated with a predefined stock-out risk of the random variable $Y_{i,t+L_i}$.

$$Y_{i,t+L_i} = \sum_{l} \sum_{h=0}^{h=L_i-1} \sum_{\mathscr{E}_i^l | \lambda_{ik} \geq H_F^l - h} a_{ik} \cdot X_{k,t+\lambda_{ik}+h}^l \quad (4)$$

In the steady state, characterized by the stability of the planning BOMs, this variable becomes Y_i and $R_{i,t+L_i}$ is replaced by R_i.

$$Y_i = \sum_{l} \sum_{h=0}^{h=L_i-1} \sum_{\mathscr{E}_i^l | \lambda_{ik} \geq H_F^l - h} a_{ik} \cdot X_k^l \quad (5)$$

Subsequently, we will work under steady state conditions to simplify the formulation; nevertheless, the adaptation to the general case is immediate. In both cases, this random variable, which serves as a reference to determine the order-up-to-level, is a weighted sum of binomial random variables whose distribution function is easy to determine by the Monte Carlo method. Formula (6) offers a generic formulation[2] of the random variable Y_i.

$$Y_i \to \sum_j w_j \times \mathscr{B}(n_j, p_j) \quad (6)$$

When the conditions of approximation by a normal distribution are met for each binomial distribution, the random variable Y_i can be approximated by a normal distribution.

$$Y_i \to \mathscr{N}\left(\mu_i = \sum_j w_j \times n_j \times p_j, \sigma_i = \sqrt{\sum_j \left[w_j \times n_j \times p_j \times (1-p_j)\right]}\right) \quad (7)$$

2.2. Costs to consider the determination of a stock-out risk in the context of emergency supplies

In the studied context, a stock-out at any level of the SC triggers an emergency procedure that prevents production stoppages. The emergency procedure assumes that the supplier is able to mobilize additional resources to promptly produce the missing units and that it is possible to shorten the lead time through rapid delivery of the missing quantities. Mobilizing an emergency procedure at a given level on the SC level prevents stock-out propagation along the downstream part of the SC.

The interval T_i between two customer orders to its supplier for a component i, is always the same when the manufacturing calendar uses only the working days. The time interval between two successive deliveries is also T_i when the lead-time L_i is constant. For organizational reasons, L_i normally corresponds to a multiple of the reference period T also called the review period in the MRP ($T_i = T$). The orders placed at the end of day t, which is equal to the difference between R_i and the projected available inventory, arrives at the beginning of the periods $t + L_i$. This schedule is intended to meet the needs of periods $t + L_i$ to $t + L_i + T - 1$.

Emergency supplies of the missing units can be analyzed in the context of an order-up-to-level policy that is characterized by an order-up-to-level R_i designed to cope with random demand according to a stock-out probability α_i. This policy generates two types of costs: costs directly incurred by the emergency supply to avoid stoppages and costs incurred by the unused units when the order is delivered, which is a consequence of using a safety stock.

First, an emergency supply may or may not generate a fixed cost c_{F_i} that is independent of the number of missing units. This cost may correspond to the payment of a special transport (charter a plane, for example) and/or the launch of exceptional production (set-up cost). An emergency supply can also generate additional variable cost c_{V_i} per missing unit. This cost can be the unit transportation cost of a logistics provider that is specialized in rapid transit and/or an increase in the direct variable production cost of a missing unit (due to overtime, for example).

In contrast, if there is no stock-out at the end of the review period and prior to receipt of a new delivery, a residual stock generates a certain cost. Each component unit i held during the review interval T generates a periodic holding cost p_i, which is calculated as the product of an annual unitary holding cost π_i and the duration T (in years).

The amount of these charges depends on the order-up-to level R_i. The minimization of the global cost of the procurement policy allows one to independently define, for each component i, the optimal order-up-to level R_i^* associated with the optimal stock-out probability $\alpha_i^* = \mathrm{P}(X > R_i^*)$. The optimal stock-out risk has no reason to be the same for all the components.

This type of inventory problem can be viewed as a variant of the newsboy problem, which introduces a lump-sum cost to pay in case of stock-out. This problem was approached by Wagner et al. (1975). Noori and Bell (1982) use an approximate formulation to resolve the periodic problem of supply of foreign currencies in a banking agency. Hill and al. (1989) were interested in the management of spare parts for equipment that reach the end of their life cycle. Aneja and Noori (1887) proposed a S,s supply model that introduces a fixed cost of support in case of shortage. Apparently, there is no existing model that addresses the emergency supply of a supply chain; the formulations in the listed bibliography are not identical to the formulations presented here.

[2] The notations used in (6) have no physical significance. They are used to obtain a generic mathematical expression of Y_i.

3. DETERMINATION AND IMPLEMENTATION OF THE OPTIMAL EMERGENCY SUPPLY POLICY

After reviewing the analytical formulation of the problem and highlighting the relationship that characterizes the optimal policy (§ 3.1), we examine the decision rule for selecting the more interesting policy for emergency supply (§ 3.2). We illustrate its application with a simple numerical example (§ 3.3).

3.1. Emergency supply model and optimal solution

The cost function for minimizing $\mathrm{C}(R_i)$, defined over the review period T, is the sum of a mathematical expectation of the holding cost $\mathrm{CP}(R_i)$ and a mathematical expectation of a stock-out cost $\mathrm{CS}(R_i)$. We use a discrete formulation of the problem, followed by a continuous formulation.

$$\mathrm{C}(R_i) = \mathrm{CP}(R_i) + \mathrm{CS}(R_i) \tag{8}$$

$$\mathrm{CP}(R_i) = p_i \times \sum_{y_i = R_i + 1}^{+\infty} (R_i - y_i) \times \mathrm{P}(Y_i = y_i) \tag{9}$$

$$\mathrm{CS}(R_i) = c_{F_i} \times \mathrm{P}(Y_i \geq R_i + 1) + c_{V_i} \times \sum_{y_i = R_i + 1}^{+\infty} (y_i - R_i) \times \mathrm{P}(Y_i = y_i) \tag{10}$$

In the continuous case, (9) becomes (11) and (10) becomes (11).

$$\mathrm{CS}(R_i) = c_{F_i} \times \mathrm{P}(Y_i \geq R_i) + c_{V_i} \int_{R_i}^{\infty} (y_i - R_i) f(y_i) dy_i \tag{11}$$

$$\mathrm{CP}(R_i) = p_i \int_{0}^{R_i} (R_i - y_i) f(y_i) dy_i \tag{12}$$

The first term $\mathrm{CP}(R_i)$ is the product of the periodic holding cost of one unit of a component i that is held during one review period and the mathematical expectation of the remaining stock at the end of the review period. The remaining stock level depends on the order-up-to level R_i and random demand Y_i of the component i.

The second term $\mathrm{CS}(R_i)$ depends on the order-up-to level R_i and random demand Y_i covered by R_i. It involves the fixed and variable costs identified previously. One of these two costs—but not both simultaneously—may be null:

- the first part of this cost, $c_{F_i} \times \mathrm{P}(Y_i \geq R_i + 1)$, is the mathematical expectation of a fixed expense that is independent of the number of missing units;
- the second part of this cost, $c_{V_i} \times \sum_{y_i = R_i + 1}^{+\infty} (y_i - R_i) \times \mathrm{P}(Y_i = y_i)$, corresponds to the mathematical expectation of the variable additional expenses generated by the expected stock-out amount.

We seek to determine the stock-out risk α_i^* associated with the order-up-to level R_i^* that minimizes the global cost $\mathrm{C}(R_i)$. In the discrete case, the two cost functions are monotone (increasing for $\mathrm{CP}(R_i)$ and decreasing for $\mathrm{CS}(R_i)$) with R_i^* satisfying the system of inequalities (13).

$$\begin{aligned} &\mathrm{C}(R_i^*) - \mathrm{C}(R_i^* + 1) < 0 \\ &\mathrm{C}(R_i^*) - \mathrm{C}(R_i^* - 1) < 0 \end{aligned} \tag{13}$$

The determination of R_i^*, and thus of α_i^*, is achieved through the study of the function $\mathrm{C}(R_i) - \mathrm{C}(R_i + 1)$. Depending on the values of c_{F_i} and c_{V_i}, evaluating this function is more or less easy to achieve. After development and replacement of $\mathrm{CP}(R_i)$ and $\mathrm{CS}(R_i)$ by (9) and (10), respectively, we obtain (14).

$$\mathrm{C}(R_i) - \mathrm{C}(R_i + 1) = c_{F_i} \times \mathrm{P}(Y_i = R_i + 1) - p_i + (c_{V_i} + p_i) \times \mathrm{P}(Y_i \geq R_i + 1) \tag{14}$$

In the continuous case, the optimum is defined by $d\mathrm{C}(R_i)/dR_i = 0$. In both cases, we distinguish three cases according to the values assumed by c_{F_i} and c_{V_i}.

3.1.1 Case 1: no fixed cost in emergency supply $(c_{F_i} = 0)$

Under these conditions, we find the classical formulation of the newsvendor problem where the optimal stock-out risk value is given by (15).

$$\mathrm{P}(Y_i \geq R_i + 1) < p_i \big/ \left(c_{V_i} + p_i\right) < \mathrm{P}(Y_i \geq R_i) \tag{15}$$

In the continuous case, we obtain (16).

$$\alpha_i^* = p_i \big/ \left(c_{V_i} + p_i\right) = 1 \big/ \left(c_{V_i} \big/ (p_i + 1)\right) \tag{16}$$

The optimal stock-out probability α_i^* depends directly on the relative cost structure c_{V_i} / p_i. The order-up-to level R_i^* is the fractile associated with α_i^*.

The inverse functions of the major probability distributions are available in spreadsheet applications for continuous and discrete distributions. As specified in (4), the demand distribution of Y_i is a weighted sum of binomial variables. When the daily production consists of several hundreds of units, this sum is generally well approximated by a normal distribution, unless the utilized probabilities of the alternative modules that require the component i are very low. Nevertheless, the exact optimal solution can be obtained after the reconstitution of the distribution function of Y_i by the Monte Carlo method.

3.1.2 Case 2: no variable cost in emergency supply $(c_{V_i} = 0)$

In this case (14) is replaced by (17).

$$\mathrm{C}(R_i) - \mathrm{C}(R_i + 1) = c_{F_i} \times \mathrm{P}(Y_i = R_i + 1) + p_i \times \mathrm{P}(Y_i < R_i + 1) \tag{17}$$

The optimality is reached when the relation (18) is satisfied.

$$\frac{P(Y_i = R_i^* + 1)}{P(Y_i < R_i^*)} < \frac{p_i}{c_{F_i}} < \frac{P(Y_i = R_i^*)}{P(Y_i < R_i^* - 1)} \tag{18}$$

In the continuous case, we obtain the relation (19) in which f is the probability density function of the random variable representing the demand[3].

$$f(R_i^*) \big/ P(Y_i < R_i^*) = p_i \big/ c_{F_i} \tag{19}$$

Whether we are in a discrete case or in a continuous case of a normal distribution, the numerical determination of the optimal solution and the creation of an abacus linking α_i^* to $p_i \big/ c_{F_i}$ is relatively simple.

When the demand Y_i is a weighted sum of binomial distributions, the solution can be obtained through the Monte Carlo simulation to obtain the probability distribution of the demand. When the normal approximation of Y_i can be realized, the resolution is much easier because it is possible to construct an abacus using a standardized normal distribution.

With $Y_i \to \mathcal{N}(\mu_{Y_i}, \sigma_{Y_i})$ and $U = (Y_i - \mu_{Y_i}) \big/ \sigma_{Y_i}$ (u is the realization of the standardized normal random variable U), the relation (19) can be replaced by (20) where $u_i^* = (R_i^* - \mu_{Y_i}) \big/ \sigma_{Y_i}$ and Φ is the cumulative distribution function of the standardized normal distribution.

$$f(u_i^*) \big/ \Phi(u_i^*) = p_i \big/ c_{F_i} \times \sigma_{Y_i} \tag{20}$$

The function $g(u) = f(u) \big/ \Phi(u)$ can be tabulated to construct a chart that gives α_i^* for different values of $p_i \big/ c_{F_i}$ and σ_{Y_i}.

3.1.3. General case: $c_{V_i} \neq 0$ and $c_{F_i} \neq 0$

This is the general case, as given by (10), where the stock-out cost is the sum of a fixed cost and a variable cost. As in the previous case, an approximation of the demand Y_i by a normal distribution can be considered to numerically attain the optimal value of the stock-out risk according to the following equation: $c_{F_i} \times f(R_i^*) = p_i - (c_{V_i} + p_i) \times P(Y_i > R_i^*)$.

This relation is equivalent to (21), obtained after standardization.

$$\frac{f(u_i^*)}{1 - \Phi(u_i^*)} = \frac{\sigma_{Y_i}}{c_{F_i}} \times \left[p_i \times \left(\frac{\Phi(u_i^*)}{1 - \Phi(u_i^*)} \right) - c_{V_i} \right] \tag{21}$$

For different values of σ_{Y_i}, curves representing α_i^* function of $p_i \big/ c_{F_i}$ and $c_{V_i} \big/ c_{F_i}$ can be drawn.

3.2. The choice between emergency supply systems

Of the three cases of emergency supply, the last one is the least common. Often, a company has to choose between the first two cases. In the first case $(c_{F_i} = 0)$, an agreement is made with a company specializing in international express freight, with a guarantee of a short delivery time and a transportation cost c_{V_i} per delivery component. In the second case $(c_{V_i} = 0)$, a means of emergency freight transportation (plane, truck), which is entirely dedicated to emergency transportation, is used; its cost c_{F_i} does not depend on the number of transported units.

In this section, we propose a simple rule to help managers to decide which case is more interesting when they have to choose between an emergency supply with a variable cost c_{V_i} and a supply solution with a fixed cost c_{F_i}.

To select the more interesting solution, let us begin with the optimal stock-out of the case 2 (α_i^{2*}) associated with the order-up-to level R_i^{2*}. In case 1, the use of this stock-out level yields a similar holding cost. We introduce $\tilde{c}_{V_i}$, the variable cost that offers the same mathematical expectation of a stock-out cost, and therefore, the same total cost for the two cases when the stock-out risk is α_i^{2*}.

$$\tilde{c}_{V_i} = c_{F_i} \Big/ \left\{ \sigma_i \left[\left(f(u_i^{2*}) \big/ P(U > u_i^{2*}) \right) - u_i^{2*} \right] \right\} \tag{22}$$

In (22) we note $u_i^{2*} = \left(R_i^{2*} - \mu_{Y_i} \right) \big/ \sigma_{Y_i}$ and $\alpha_i^{2*} = P(Y_i > R_i^{2*})$.

By analogy, we write $\tilde{c}_{F_i}$, the fixed cost that offers the same mathematical expectation of a stock-out cost, and therefore, the same total cost for the two cases when the stock-out risk is α_i^{1*}.

A simple rule that covers the majority of the cases is formulated as following:

- the case 1 is better than the case 2 when $c_{V_i} < \tilde{c}_{V_i}$;
- the case 2 is better than the case 1 when $c_{F_i} < \tilde{c}_{F_i}$.

3.3 Numerical example

Let us now illustrate numerically the calculation of the optimal stock-out risk α_i^* for a component i in the first two cases mentioned above.

In this part, we develop the numerical example presented in (Giard and Sali, 2012) where the procurement of piston crowns for automotive assembly plants is considered.

A unit purchasing cost $UPC_i = 100$ € and a weekly holding rate $\pi_i = 0{,}29\%$ are utilized to calculate a periodic holding cost for one piston crown $p_i = 0{,}29$ €.

[3] If the density function is symmetrical, as it is for the Normal Distribution, $P(Y_i < R_i) = P(Y_i > 2\overline{Y}_i - R_i)$, $f(R_i) = f(2\overline{Y}_i - R_i)$ and $f(R_i) / P(Y_i < R_i) = f(2\overline{Y}_i - R_i) / P(Y_i > 2\overline{Y}_i - R_i)$, which is the definition of the hazard distribution.

In (Giard and Sali, 2012), the application of the MRP mechanism, as discussed in §2, provides a demand Y_i for this component following a weighted sum of binomial random variables.

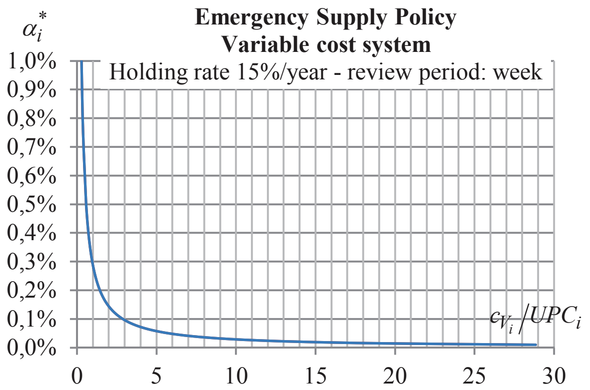
$$Y_i \rightarrow 4\times\mathscr{B}(960,0.2)+4\times\mathscr{B}(1840,0.54)+ \\ 4\times\mathscr{B}(960,0.2)+6\times\mathscr{B}(960,0.1) \quad (23)$$

The normal approximation of Y_i allows us to write (24).

$$Y_i \rightarrow \mathscr{N}(6086.4, 123.84) \quad (24)$$

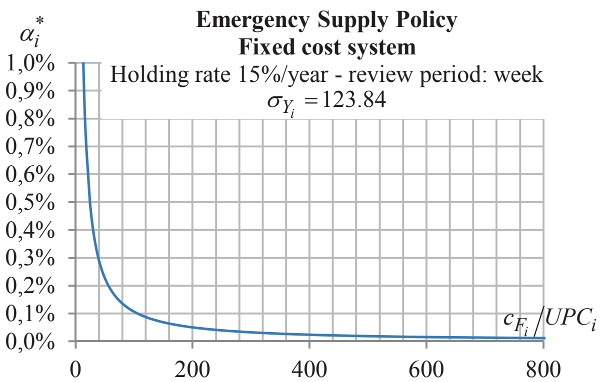
3.3.1. Case 1: no fixed cost in emergency supply $(c_{F_i}=0)$

When no fixed cost is considered, the calculation of α_i^* depends on the relative cost structure c_{V_i}/p_i. Using a multiplicative constant, this ratio is equivalent to the ratio of the variable cost of emergency supply and the Unit Purchasing Cost UPC_i, as shown below.



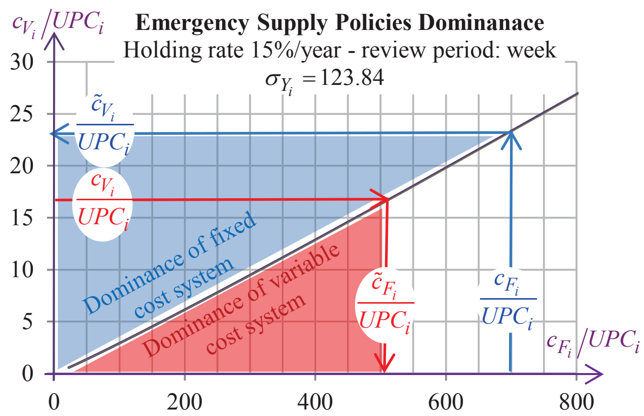
3.3.2. Case 2: no variable cost in emergency supply $(c_{V_i}=0)$

In the second case, the use of the hazard function, after a normal approximation of the demand Y_i by (24), is required to link α_i^* to c_{F_i}/UPC_i, which yields the following curve.



3.3.3. Comparison of relative dominance in policies of emergency supply where $c_{F_i}=0$ *or* $c_{V_i}=0$

With $c_{F_i}=10600$ we obtain $\alpha_i^*=0.1\%$. To attain the same total expected cost with the alternative policy, the variable emergency supply cost must be $\tilde{c}_{V_i}=309$. For any $c_{V_i}<\tilde{c}_{V_i}$, the variable cost policy gives a better economic performance.



4. CONCLUSIONS

We have demonstrated how it is possible to determine the optimal stock-out risk in the case of emergency supply. This article represents the continuation of previous work on the design of a procurement policy in the context of mass production of highly diversified products.

We have addressed two common cases of emergency supply in which the stock-out cost is the sum of a fixed cost and a variable cost depending on the amount of component to supply.

As shown by the numerical example, simple abacus can be constructed, using a normal approximation of the demand, to assist operational decision makers

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